Exercise 71

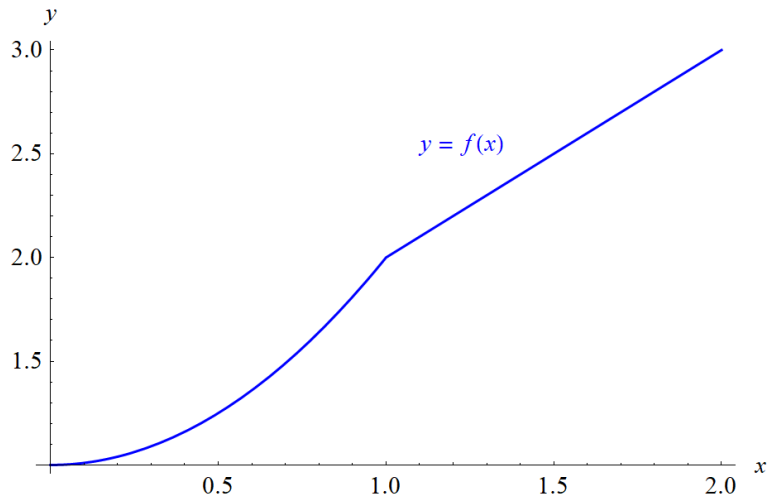
Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$$

Is f differentiable at 1? Sketch the graphs of f and f'.

Solution

Below is a graph of $f(x)$ versus x.



Although the function is continuous, there's a kink in the curve at $x = 1$, which means its slope (or derivative) is undefined there. That is, f is not differentiable at 1. The derivative of f is

$$f'(x) = \begin{cases} 2x & \text{if } x < 1 \\ 1 & \text{if } x > 1 \end{cases},$$

and its graph versus x is shown below.